Learning physics means learning how to deploy a small set of general principles flexibly in a variety of problem situations. But how is this problem-solving flexibility learned? Can standard instructional methods be modified to enhance this learning? Physics education research provides a method for seeking answers to these types of questions empirically, by pairing investigations into student thinking with instructional design and assessment. In this tradition, I will present a trajectory of research investigating problem-solving flexibility in physics. In-depth investigations of student thinking have identified ways in which students’ problem-solving approaches are rigid and fail to capitalize on opportunities for efficient and insightful problem solving. These findings informed the development of instructional methods focused on teaching and assessing problem-solving flexibility. The instructional approach emphasized the development of robust connections between conceptual and mathematical reasoning, as it was hypothesized that a lack of coherence between the two was at the root of students’ rigid problem solving. An empirical comparison in introductory, calculus-based physics courses showed that students in this coherence-focused course were more likely to employ flexible, effective, and insightful problem-solving approaches than students in a traditionally-taught course. I will conclude by laying out the roadmap for future theoretical and instructional directions in teaching problem-solving flexibility.