Your department chair (or advisor) called…

…to say that you will be teaching a class next semester that you have never taught before.

What do you want to know so you can plan and offer an effective course? (Alternative phrasing: If you HAD taught the course before, what useful knowledge would you already possess?)
Why did they think that? The use and development of mathematical knowledge for teaching at the undergraduate level

Natasha Speer
Department of Mathematics & Statistics
Center for Research in STEM Education
The University of Maine
Your department chair (or advisor) called…

…to say that you will be teaching a class next semester that you have never taught before.

What do you want to know so you can plan and offer an effective course? (Alternative phrasing: If you HAD taught the course before, what useful knowledge would you already possess?)

Discuss this with the people seated near you. We will share out ideas in a few minutes.
Today

• Overview of research on knowledge used in teaching mathematics

• Connections to improving teaching and learning of undergraduate mathematics

• Example of research designed to inform these efforts

• Implications for supporting novice instructors and further research

… with “work of teaching” tasks scattered throughout
Domains of Mathematical Knowledge for Teaching

SUBJECT MATTER KNOWLEDGE

- Common content knowledge (CCK)
- Horizon content knowledge
- Specialized content knowledge (SCK)

PEDAGOGICAL CONTENT KNOWLEDGE

- Knowledge of content and students (KCS)
- Knowledge of content and teaching (KCT)
- Knowledge of content and curriculum

Ball, Hoover Thames, & Phelps (2008)
Domains of Mathematical Knowledge for Teaching

- Subject Matter Knowledge
  - Common content knowledge (CCK)
  - Horizon content knowledge
  - Specialized content knowledge (SCK)

- Pedagogical Content Knowledge
  - Knowledge of content and students (KCS)
  - Knowledge of content and teaching (KCT)
  - Knowledge of content and curriculum

Ball, Hoover Thames, & Phelps (2008)
Common Content Knowledge

"knowledge of a kind used in a wide variety of settings – in other words not unique to teaching"; these are not specialized understandings but are questions that typically would be answerable by others who know mathematics” (Ball, Hoover Thames, & Phelps, 2008, p. 399)

$$472 - 295 = 177$$

If $$f(x) = 3\sin(x)$$,

$$y' = 3\cos(x)$$
Knowledge used to do mathematics

• Necessary, but not sufficient for teaching (Ball & Bass, 2000; Hill, Rowan, & Ball, 2005; Hill, Sleep, Lewis, & Ball, D., 2007; Monk, 1994)

• More courses in content are not strongly correlated with higher student achievement (Begle, 1979; Monk, 1994)
Domains of Mathematical Knowledge for Teaching

**SUBJECT MATTER KNOWLEDGE**
- Common content knowledge (CCK)
- Horizon content knowledge

**PEDAGOGICAL CONTENT KNOWLEDGE**
- Specialized content knowledge (SCK)
- Knowledge of content and students (KCS)
- Knowledge of content and teaching (KCT)
- Knowledge of content and curriculum

Ball, Hoover Thames, & Phelps (2008)
Domains of Mathematical Knowledge for Teaching

SUBJECT MATTER KNOWLEDGE

- Common content knowledge (CCK)
- Horizon content knowledge

PEDAGOGICAL CONTENT KNOWLEDGE

- Knowledge of content and students (KCS)
- Knowledge of content and teaching (KCT)
- Knowledge of content and curriculum

Ball, Hoover Thames, & Phelps (2008)
Teaching task #1

On a pre-calculus test, students need to expand the following expression:

\[(x + 3)^2\]

- What are some answers students might produce?
- What might students be thinking as they produce the incorrect answers?
Pedagogical Content Knowledge (PCK)

Typical ways students think (productively and unproductively), common student strategies, especially useful examples, organization of content in a course, relative difficulty of topics

(Shulman, 1986)

– influences practices

(Carpenter et al., 1988, 1989)

– influences student learning
  (e.g., Fennema et al., 1996; Franke et al., 2001; 1997)
Domains of Mathematical Knowledge for Teaching

SUBJECT MATTER KNOWLEDGE

Common content knowledge (CCK)

Horizon content knowledge

Specialized content knowledge (SCK)

PEDAGOGICAL CONTENT KNOWLEDGE

Knowledge of content and students (KCS)

Knowledge of content and teaching (KCT)

Knowledge of content and curriculum

Ball, Hoover Thames, & Phelps (2008)
This question was on a calculus quiz:

If \( y = (5 + x^2)^7 \), what is the derivative of \( y \)?

Several students gave this answer:

\[
y' = 7(5 + x^2)^6 (2x) (2)
\]

What might the students have been thinking?

Bonus problem: \[
\begin{array}{c}
472 \\
- 295
\end{array}
\]

What’s the thinking here?
Specialized Content Knowledge (SCK)

- Used to
  - do the “mathematical work” of teaching
  - follow and understand students’ mathematical thinking
  - evaluate the validity of student-generated strategies

- Shown to play a role in teachers’ practices and correlate with students’ learning
  (Ball & Bass, 2000; Hill et al 2004, 2005; Ma, 1999)

Is important and distinct from “common” content knowledge
...and that class you will be teaching for the first time uses collaborative groupwork, inquiry-based instruction and other active learning approaches.

• Are there types of knowledge that would be especially important/valuable for this instructional approach?

• What are some types of knowledge that might be important in this situation that would be less crucial in an all-lecture format?

Discuss.
Using “engaged student learning” approaches

• is challenging
  – requires facilitating discussion and progression of ideas (Stein et al., 2006)
  – often means adopting new practices

• places different demands on instructors’ knowledge than traditional lecture (Speer & Wagner, 2009; Wagner, Speer & Rosa, 2007)

So, why should we bother?
To have an adequate number of people for STEM jobs, we (universities, colleges) need to produce 1 million ADDITIONAL graduates with STEM degrees over the next decade.

(Holdren & Lander, 2012)
Part of that reality

What percent of freshmen who intend to major in STEM (science, technology, engineering, mathematics) actually finish a STEM major?

(1) > 80%
(2) 60 – 80%
(3) 40 – 60 %
(4) < 40 %
Part of that reality

What percent of freshmen who *intend* to major in STEM (science, technology, engineering, mathematics) *actually* finish a STEM major?

(1) > 80%
(2) 60 – 80%
(3) 40 – 60%
(4) < 40 %
“The first two years of college are the most critical to retention and recruitment of STEM majors.”

Holdren & Lander, 2012, p. 6
“The lecture has been a mainstay of higher education since the word ‘lecture’ was created in the 14th century, and today most introductory STEM courses are taught largely through lectures.”

“Extensive research on how the human brain learns indicates that diversifying teaching methods enhances critical thinking skills, long-term retention of information, and student retention in STEM majors.”

Holdren & Lander, 2012, p. 8
Evidence from a meta-analysis*

- 200 studies of undergraduate STEM instruction
- Two categories:
  - passive lecture
  - anything else
- Passive lecture courses: 55% higher DFW rates
- Medical metaphor

“...these active learning techniques benefit all students and can close the achievement gap between ethnic groups and men and women” (Holdren & Lander, 2012, p. 9)
Minorities in STEM

% of Degrees to Under-rep. Minorities

- College-age pop: 36%
- US pop: 30%

Graph showing the percentage of degrees awarded to under-represented minorities in various STEM fields from 1987 to 2012.
In mathematics

- Inquiry-based learning (IBL) in over 100 course sections at 4 universities
- Despite variation in how IBL was implemented, student outcomes are better relative to traditionally taught courses
- “The use of IBL eliminates a sizable gender gap that disfavors women students in lecture-based courses.”

In calculus

- While taking calculus, some students will decide to switch out of a major that requires the calculus sequence.
- Female students are 1.5 times as likely to decide that as their male classmates (with the same preparedness, career goals, etc.).

What makes it difficult?

“A significant barrier to broad implementation of evidence-based teaching approaches is that most faculty lack experience using these methods and are unfamiliar with the vast body of research indicating their impact on learning” (Holdren & Lander, 2012, p. iii).
What makes it difficult?

Preparing the next generation of instructors (graduate students):

• Not always possible to provide substantial pre-service preparation

• Professional development concurrent with teaching sometimes limited

• TA professional development is not typically a primary focus for faculty
Rest of this talk

• Sample of study examining college instructor MKT

• What knowledge of student thinking do experienced instructors possess?
  – And how does that compare to novices?

• Findings can inform design of professional development for novice instructors
Research design

Context: Calculus
Task-based interviews

Seven tasks borrowed from or modeled after those used in research on student thinking (e.g., Habre & Abboud, 2006; Kendal & Stacey, 2003; White & Mitchelmore, 1996; Zandieh, 2000).

Instructors:

• research mathematicians recognized for teaching excellence

• graduate students with fewer than two years of calculus teaching experience
PART 1: Solve task and describe solution

PART 2: Describe how students would solve the task, including difficulties/mistakes they might make and correct/incorrect ways of thinking they might display

PART 3: Examine and discuss student work samples (that represent typical student difficulties, ways of thinking, etc.) Based on protocol from previous studies (Frank & Speer, 2011, 2012; Speer et al., 2006)

PART 4: Describe what you would do/say to student who produced the work during office hours

Approximately 20 minutes per task.
Four-PART INTERVIEW

PART 1: Solve task and describe solution

PART 2: Describe how students would solve the task, including difficulties/mistakes they might make and correct/incorrect ways of thinking they might display

PART 3: Examine and discuss student work samples (that represent typical student difficulties, ways of thinking, etc.) Based on protocol from previous studies (Frank & Speer, 2011, 2012; Speer et al., 2006)

PART 4: Describe what you would do/say to student who produced the work during office hours

Approximately 20 minutes per task.
The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are travelling in the same direction.)

**Question**: State the relationship between the position of car A and car B at $t = 1$ hour. Provide an explanation for your answer.

From Carlson (1998)
Experience the interview

The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are travelling in the same direction.)

Question: State the relationship between the position of car A and car B at t = 1 hour. Provide an explanation for your answer.

- With a neighbor, discuss:
  - Your thinking and answer to the question
  - How students might solve the problem
  - Typical student productive and unproductive ways of thinking
Ways of thinking

1. Distance car A traveled > Distance car B traveled
2. Distance car A traveled = Distance car B traveled
3. Distance car A traveled < Distance car B traveled

The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are travelling in the same direction.)

**Question:** State the relationship between the position of car A and car B at t = 1 hour. Provide an explanation for your answer.
Ways of thinking

1. Distance car A traveled > Distance car B traveled
2. Distance car A traveled = Distance car B traveled
3. Distance car A traveled < Distance car B traveled

The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are travelling in the same direction.)

Question: State the relationship between the position of car A and car B at $t = 1$ hour. Provide an explanation for your answer.
Common ways of thinking

• Productive:
  – Use relative speed
  – Use the relative areas under the curves

• Unproductive:
  – Interpret the graphs literally as the paths of the cars, rather than interpreting the functional information displayed on the graph
    • The cars will collide at one hour
    • “Two different paths to get to the same place”
  – Interpret the speed graphs as position graphs
  – Compare function slopes at \( t = 1 \)
Examining student work

Student A:

Car A and Car B probably cross each other at 1 hour because there might be 2 roads to take.

Student B:

They are in the same place even though at first A was going much faster, but B later increased and caught up.

Student F:

At $t = 1$ the cars are going at the same speed but car A traveled much farther because it had a higher velocity for a longer time.
Research design: Analysis methods

• Findings from research on student thinking were used to characterize the extent to which participants were knowledgeable of student thinking

• Methods from Grounded Theory (Strauss & Corbin, 1990) were also used
  • to identify themes
  • to detect comparisons/contrasts
Brief overview of findings

• All participants gave correct solutions to the task (some took a little while…).

• All participants described some **productive** ways of thinking that students might use

• All participants gave some possible **unproductive** ways of thinking
Findings about the mathematicians

• generated more possible ways of thinking
• used their (more extensive) set of ways of thinking to make sense of the written work
• PLUS: they were able to figure out (new-to-them) ways of thinking from examining student work

They learned during the interview!

• They did more (spontaneous) quantifying/calibrating of how common a particular difficulty might be
  – “…a lot of students probably would…”
  – “I would expect many to get it wrong”
  – “Some students might say…”
Findings about the graduate students

- Most generated very few possible ways of thinking
- Some answers were fairly general:
  - “[they] don’t know how to read this graph I think…so we have this speed and we have time but how can I, how do you represent the distance based on this graph?” (A, graduate student)
- Some focused on small variations to one approach, mostly the way they thought about the problem:
  - “…So they might make the same mistake as I was by looking at the slope because for some reasons I keep thinking of this as a position instead of speed…” (C, graduate student)
Closing thoughts

• Faculty gain MKT from their experiences
• MKT supports effective instruction
• Need to leverage this to improve student learning

• How I (and others) use these findings
  – Design professional development for novice college mathematics instructors
    • access to known student ways of thinking
    • opportunities to learn from examining student work
  – Equip the next generation of faculty with knowledge needed to enact active, engaged student learning approaches
  – Work toward improved learning, retention, access and opportunity for all students
Thank you!

Questions and/or comments?

Acknowledgements

my email: speer@math.umaine.edu
The Master of Science in Teaching (MST) Program at the University of Maine

- Study math &/or science content and teaching
- Obtain teaching experience
- Conduct original research with faculty

www.umaine.edu/center
Overview of the MST program

• 2-year master’s program offered through the Maine Center for Research in STEM Education (RiSE Center)

• Research & certification opportunities in life science, physical science, earth science, and mathematics

• Unique combination of coursework, thesis, and teaching experience

• Paid assistantships for full time students

• **NSF Teaching Fellowship Program** for selected MST graduates who choose to teach in high-need Maine districts ($10,000 per year)

www.umaine.edu/center


References


